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Determination of Electron Density in Plasmas from the Hydrogen Spectral Line H_α Broadened by Combined Stark and Zeeman Effect

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(Z. Naturforsch. **28 a**, 789–791 [1973]; received 3 February 1973)

A method is described which permits a rapid determination of the electron density in plasmas from the spectral line H_α broadened by combined Stark and Zeeman effect.

A study of the influence of an intense magnetic field on the Stark profiles of spectral lines emitted by magnetically confined hydrogen plasmas has been undertaken theoretically as well as experimentally [1–5]. A magnetic field of strength $\boldsymbol{H}$ has the following essential effects on Stark-broadened spectral lines:

1. Partial polarization of the emitted light. The relative contribution of the Zeeman components of intensities I_σ and I_π to the total line profile depends in a sensitive manner on the angle Θ_0 which has the direction of observation $\boldsymbol{k}$ relative to the direction of $\boldsymbol{H}$. We have shown [2] that the intensity $I_{\boldsymbol{k}}(\Delta\lambda)$ within the observed line in direction $\boldsymbol{k}$ is given by

$$I_{\boldsymbol{k}}(\Delta\lambda) = I_{\parallel}(\Delta\lambda)\cos^2\Theta_0 + I_{\perp}(\Delta\lambda)\sin^2\Theta_0 \quad (1)$$

where $I_{\parallel}(\Delta\lambda)$ and $I_{\perp}(\Delta\lambda)$ are respectively the intensities within the lines observed longitudinally ($\parallel$) and transversally ($\perp$).

2. Magnetic splitting of the atomic sub-levels in addition to Stark splitting caused by the quasi-static electric ion field of strength $\boldsymbol{F}$. The atomic states $|\Phi(n,i)\rangle$ (with $i=1, 2, \ldots$) which intervene in the calculation of the line profiles are not further eigenstates $|n, n_1, n_2\rangle$ of the Stark effect but rather a linear combination of states $|n, l, m\rangle$ obtained from a diagonalization of the interaction potential V_{static} given by

$$V_{\text{static}} = (e/2\,m\,c)\,\boldsymbol{H}\cdot(\mathbf{L}+2\,\mathbf{S}) + e\,\boldsymbol{F}\cdot\boldsymbol{r} \quad (2)$$

where $\mathbf{L}$ and $\mathbf{S}$ denote the angular momentum and spin vectors respectively. Putting $|\boldsymbol{F}|$ equal to the normal field strength $F_0 = 2.603\,e\,N^{2/3}$, the quantity

$$\tau = \overline{\Delta\omega}_{\text{Stark}}/\overline{\Delta\omega}_{\text{Zeeman}} = 5.15\cdot10^{-7}\,n\,N^{2/3}/H \quad (3)$$

is a measure of the relative importance of the two terms of the r.h.s. of Equation (2). (n = principal quantum number, N = electron density in cm^{-3}, H = magnetic field strength in Gauss.)

3. Bending of the electron trajectories into helical paths around the magnetic lines of force [5]. This effect can be neglected, however, as long as the Debye length r_{D} which is a measure for the shielding effect remains smaller than the mean gyration radius r_{g} of the electrons, i. e. if

$$r_{\text{D}}/r_{\text{g}} = 2.2\cdot10^2\,H\,N^{-1/2} < 1 \quad (4)$$

(N in cm^{-3}, H in G).

Noting that

$$\Delta\omega_{\text{Zeeman}}/\Delta\omega_{\text{p}} = [(n-1)/2]\,(r_{\text{D}}/r_{\text{g}}) \quad (5)$$

where ω_{p} is the plasma frequency, condition (4) permits the calculation of the broadening due to electron collisions independent of the magnetic field according to the assumption of classical straight paths and the hypothesis of completely degenerate sub-levels, except for very high quantum numbers n.

Details of the calculations have been given in [2]. The results presented here contain additionally the contribution of the perturbation of the lower level ($n'=2$) to the broadening effect. The collision operator

$$\varphi^{(n\,n')} \propto \frac{1}{a_0^2}\left(\boldsymbol{r}^{*(n')}\cdot\boldsymbol{r}^{*(n')} - 2\,\boldsymbol{r}^{*(n')}\cdot\boldsymbol{r}^{(n)} + \boldsymbol{r}^{(n)}\cdot\boldsymbol{r}^{(n)}\right) \quad (6)$$

is treated in the way that the term $-2\,\boldsymbol{r}^{*(n')}\cdot\boldsymbol{r}^{(n)}$ is considered as a weak correction compared to the expression $\boldsymbol{r}^{*(n')}\cdot\boldsymbol{r}^{*(n')} + \boldsymbol{r}^{(n)}\cdot\boldsymbol{r}^{(n)}$ which is diagonal with respect to the spherical wave functions $|n\,l\,m\rangle\,|n'\,l'\,m'\rangle$. The numerical calculations show that it suffices to retain only terms of first order.

In Figs. 1 and 2 we show as an example the calculated line profiles $I_{\parallel}(\Delta\lambda)$ and $I_{\perp}(\Delta\lambda)$ of H_α for the electron temperature $T_{\text{e}} = 2\cdot10^4\,^\circ\text{K}$ and the magnetic field strength $H = 5\cdot10^4$ G, with N as a parameter. The profiles are normalized according to

$$\int I_{\parallel}(\Delta\lambda)\,\text{d}(\Delta\lambda) = 1\,, \qquad \int I_{\perp}(\Delta\lambda)\,\text{d}(\Delta\lambda) = 1\,.$$

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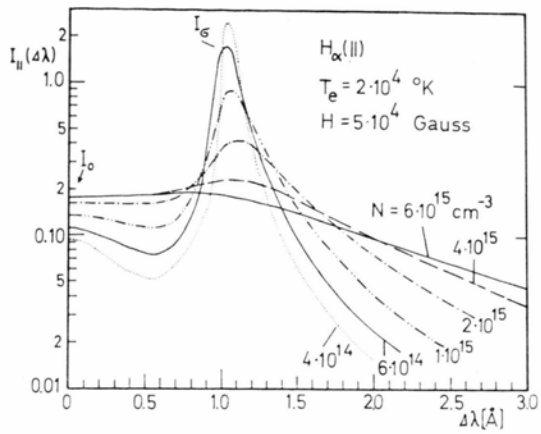

Fig. 1. Theoretical line profiles of H_α for $T_e = 2\cdot10^4$ °K and $H = 50$ kG, with N as a parameter. Observation parallel to $\boldsymbol{H}$.

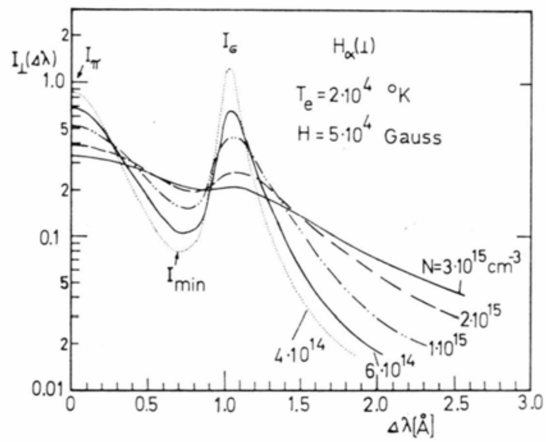

Fig. 2. Theoretical line profiles of H_α for $T_e = 2.10^4$ °K and $H = 50$ kG, with N as a parameter. Observation perpendicular to the magnetic field direction.

The theoretical as well as the experimental results show that the line profiles coincide with the pure Stark profiles if $\tau \gg 1$ and deviate progressively from these profiles as τ tends towards unity. In the region $\tau \approx 1$ the profiles broadened by combined Stark-Zeeman effect are complicated functions of N and H. When in this region line profiles are used for a determination of the electron density one has to compare the whole measured profile with the whole theoretical profile in order to obtain N.

For $\tau < 1$, however, the situation changes. The Zeeman components begin clearly to appear within the profile and the line contour changes in a characteristic and sensitive manner with electron density. Especially for H_α, $\tau < 1$ corresponds to electron densities

$$N < N(\tau = 1) = 0.5215 \cdot 10^9 \, H^{3/2} \tag{7}$$

(N in cm^{-3}, H in G). Inspection of the line profiles shows that the intensity $I_0 = I_{\parallel}(\Delta\lambda = 0)$ in the center of profile $I_{\parallel}(\Delta\lambda)$ and the intensity I_{min} of the minimum between the components I_π and I_σ of profile $I_\perp(\Delta\lambda)$ decrease rapidly with decreasing N whereas the intensity of the component I_σ varies in the opposite sense. For $\tau < 1$, the ratios I_0/I_σ and I_{min}/I_σ become sensitive functions of the electron density.

From our theoretical profiles we obtain directly the quantities

$V_1 = I_0/I_\sigma$ for lontitudinal observation $[I_{\parallel}(\Delta\lambda)]$,

$V_2 = I_{min}/I_\sigma$ for transversal observation $[I_\perp(\Delta\lambda)]$.

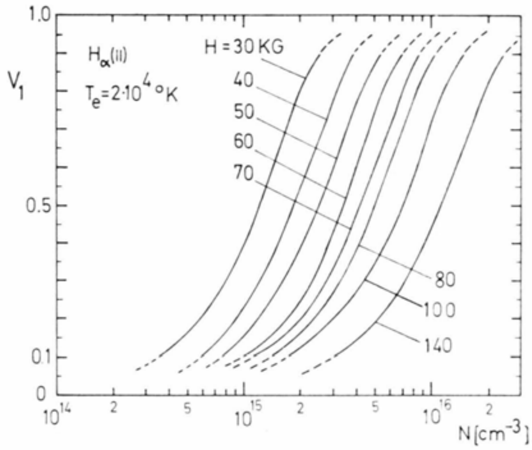

Fig. 3. Graphical representation of the quantity V_1 for $T_e = 2\cdot10^4$ °K.

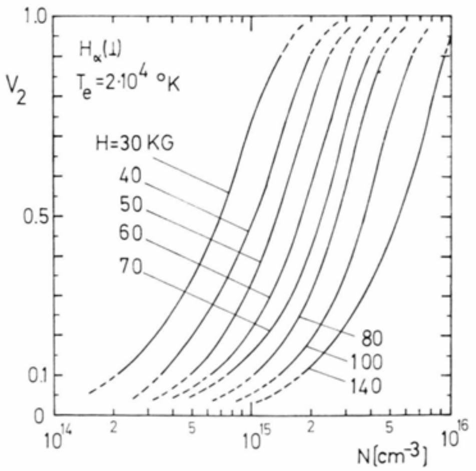

Fig. 4. Graphical representation of the quantity V_2 for $T_e = 2\cdot10^4$ °K.

Figures 3 and 4 show the numerical values in graphical form for $T_e = 2 \cdot 10^4$ °K and different values of H. Figure 5 shows the temperature dependence of V_1.

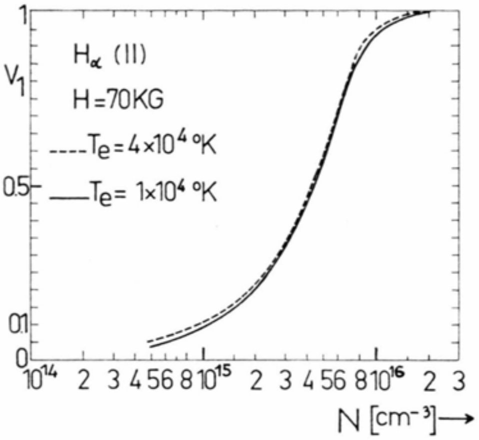

Fig. 5. The quantity V_1 for two different electron temperatures.

The Fig. 3 shows that for given values of electron temperature T_e and magnetic field strength H a direct measurement of V_1 permits a rapid and precise determination of the electron density N in the region $N(\min) \leqq N < N(\tau = 1)$, where $N(\tau = 1)$ is given by Equation (7). Numerical values for the H_α line may be found in Table 1. The value of $N(\min)$ is defined by the condition $V_1 = 0.10$.

Table 1. Numerical values of the critical electron density $N(\tau = 1)$ for the H_α line.

H[kG]	30	40	50	60	70	80	100	140
10^{-15} N (τ=1) [cm^{-3}]	2.71	4.17	5.83	7.66	9.66	11.8	16.5	27.1

In the same way it is possible to use measured values V_2 for a determination of the electron density according to Figure 4. However, due to the relatively strong contribution of the central I_π-component, the minimum intensity $I_{\min}$ varies for $N \approx N(\tau = 1)$ less rapid with N than I_0 does. It follows from this that in the region $\tau \approx 1$ V_2 has a weaker N-dependence than V_1. We have therefore to introduce another upper limit of applicability which can conveniently be expressed by the condition $N < N(\max)$, where $N(\max)$ is defined by $V_2 = 0.90$.

When in the case of transversal observation the central I_π-component is suppressed by means of a polarizer one obtains a line contour which equals the $I_{||}(\Delta\lambda)$-profile for longitudinal observation. It is then possible to use V_2-values for a determination of N.

New calculations based on more recent expressions for the electronic collision operator and on the O(4)-symmetry properties of the wave functions in the diagonalization of the interaction potential Eq. (2) as well as experimental verifications of the theoretical calculations are in progress.

We emphasize that the method described here is not restricted to the H_α line. One can also use other spectral lines such as H_β, H_γ . . . provided the theoretical profiles are known. For a given magnetic field strength each line has a characteristic range of electron density for which the corresponding values I_0/I_σ and $I_{\min}/I_\sigma$ respectively become a sensitive function of N.

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